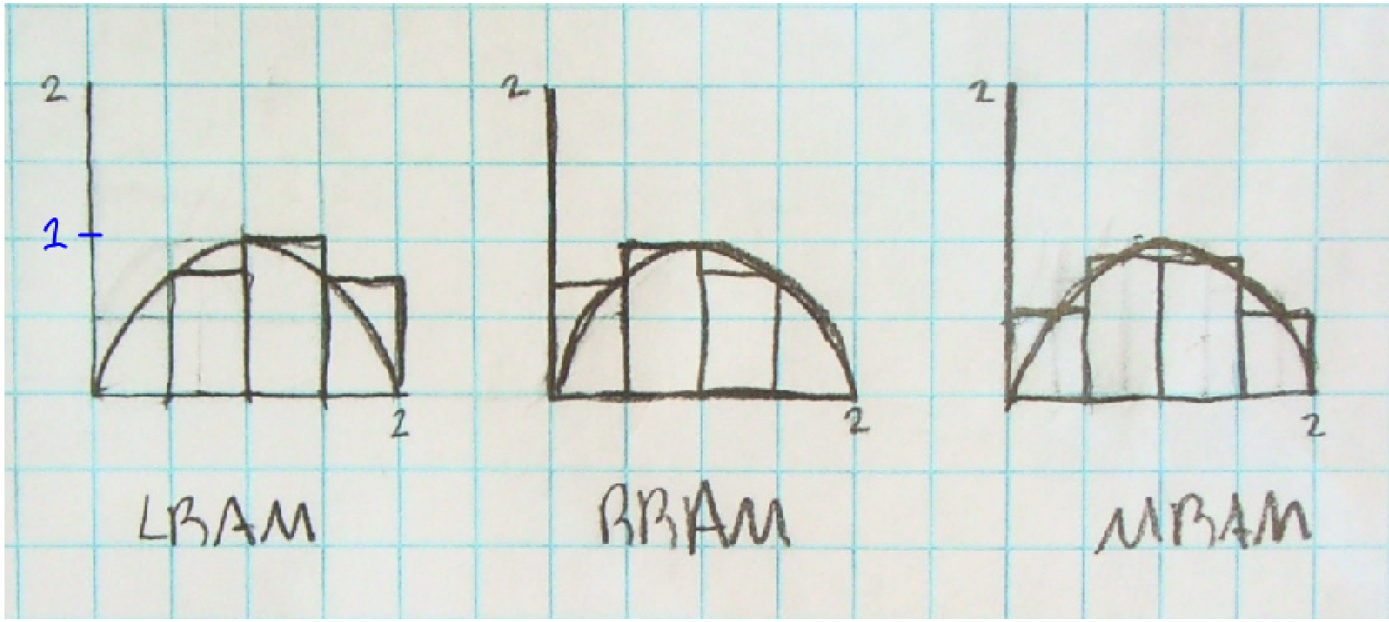
# No Calculator

① The region R is enclosed between the graph of the function $y=2x-x^2$ and the x-axis. Sketch the region for $0 \le x \le 2$ and divide it into four subintervals. Sketch the rectangles used to compute the LRAM, RRAM, and MRAM and find the area under the curve with each.

② Evaluate

$$\int_a^b 3t\,dt \quad , \quad 0<a<b$$




$\frac{1}{2}\left(0 + \frac{3}{4} + 1 + \frac{3}{4}\right)$

$= 1.25$

$\frac{1}{2}\left(\frac{3}{4} + 1 + \frac{3}{4} + 0\right)$

$= 1.25$

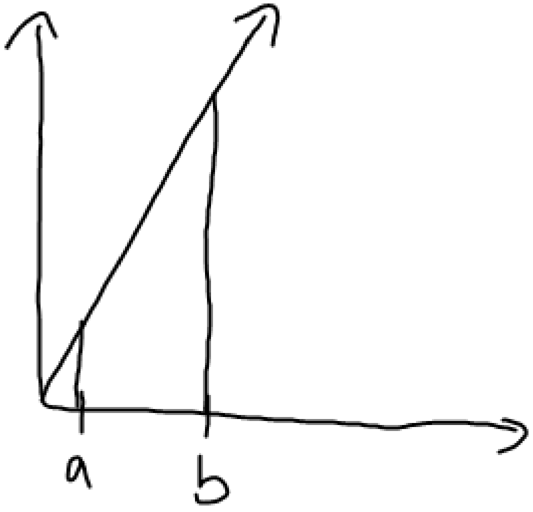
$\frac{1}{2}\left(0.437 + 0.9375 + 0.9375 + 0.437\right)$

$\approx 1.375$

$$\int_0^2 (2x - x^2)\,dx = \left(2^2 - \frac{2^3}{3}\right) - \left(0^2 - \frac{0^3}{3}\right) = \frac{4}{3} \approx 1.\overline{33}$$

$$F(x) = x^2 - \frac{x^3}{3}$$

$$\int_a^b 3t\,dt$$




$f(a)$

$3a$

$h=(b-a)$

$b_1=3a$

$b_2=3b$

$$A=\frac{1}{2}(b_1+b_2)h$$

$$\frac{1}{2}(3a+3b)(b-a)$$

$$\frac{3}{2}(a+b)(b-a)$$

$$\frac{3}{2}(b^2-a^2)$$

$$\int_a^b 3t\,dt$$

$$\cancel{t^{n+1}} \qquad \frac{3}{2}t^2$$

$$\frac{3}{2}b^2 - \frac{3}{2}a^2$$

# Fundamental Theorem of Calculus

① $\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$

② $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is the antiderivative

$= \text{Number}$

Definite integral

**EXAMPLE 2 The Fundamental Theorem with the Chain Rule**

Find $dy/dx$ if $y = \int_1^{x^2} \cos t\, dt$.

**SOLUTION**

The upper limit of integration is not $x$ but $x^2$. This makes $y$ a composite of

$$y = \int_1^u \cos t\, dt \quad \text{and} \quad u = x^2.$$

We must therefore apply the Chain Rule when finding $dy/dx$.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\
&= \left( \frac{d}{du} \int_1^u \cos t\, dt \right) \cdot \frac{du}{dx} \\
&= \cos u \cdot \frac{du}{dx} \\
&= \cos(x^2) \cdot 2x \\
&= 2x \cos x^2
\end{aligned}$$

*Now try Exercise 9.*

## EXAMPLE 3 Variable Lower Limits of Integration

Find $dy/dx$.

**(a)** $y = \int_x^5 3t \sin t\, dt$ **(b)** $y = \int_{2x}^{x^2} \frac{1}{2 + e^t}\, dt$

**SOLUTION**

The rules for integrals set these up for the Fundamental Theorem.

**(a)**
$$\frac{d}{dx}\int_x^5 3t \sin t\, dt = \frac{d}{dx}\left(-\int_5^x 3t \sin t\, dt\right)$$
$$= -\frac{d}{dx}\int_5^x 3t \sin t\, dt$$
$$= -3x \sin x$$

**(b)**
$$\frac{d}{dx}\int_{2x}^{x^2} \frac{1}{2 + e^t}\, dt = \frac{d}{dx}\left(\int_0^{x^2} \frac{1}{2 + e^t}\, dt - \int_0^{2x} \frac{1}{2 + e^t}\, dt\right)$$
$$= \frac{1}{2 + e^{x^2}}\frac{d}{dx}(x^2) - \frac{1}{2 + e^{2x}}\frac{d}{dx}(2x) \qquad \text{Chain Rule}$$
$$= \frac{1}{2 + e^{x^2}} \cdot 2x - \frac{1}{2 + e^{2x}} \cdot 2$$
$$= \frac{2x}{2 + e^{x^2}} - \frac{2}{2 + e^{2x}}$$

*Now try Exercise 19.*

**EXAMPLE 4** **Constructing a Function with a Given Derivative and Value**

Find a function $y = f(x)$ with derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$.

$$\frac{d}{dx}\int_3^x \tan t\,dt + 5$$

**SOLUTION**

The Fundamental Theorem makes it easy to construct a function with derivative $\tan x$:

$$y = \int_3^x \tan t\,dt.$$

Since $y(3) = 0$, we have only to add 5 to this function to construct one with derivative $\tan x$ whose value at $x = 3$ is 5:

$$f(x) = \int_3^x \tan t\,dt + 5.$$

*Now try Exercise 25.*

## Practice - find $\frac{dy}{dx}$

ⓐ $y = \int_2^x \left(3t + \cos(t^2)\right) dt$

$= 3x + \cos x^2$

ⓑ $y = \int_0^{x^2} e^{t^2} \quad = 2x e^{x^4}$

ⓒ $y = \int_6^{x^2} \cot(3t)\, dt$

$= 2x \cot(3x^2)$

ⓓ $y = \int_x^6 \ln(1+t^2)\, dt \quad = -\ln(1+x^2)$

ⓔ $y = \int_{3x^2}^{10} \ln(2+t^2)\, dt$

$= -6x \ln(2+9x^4)$

$\boxed{\text{HW}}$

Sect. 5.4 #1-20(5), 21-26(3), 37-40